International Journal of Theoretical Physics, Vol. 13, No. 1 (1975), pp. 27–35

The Interpretation of the Theory of Relativity

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Received: 20 *September* 1974

Abstract

The presently accepted interpretation of the (Special) Theory of Relativity is considered by the author to be inadequate in certain respects. An alternative interpretation is suggested

1. *Introduction*

The presently accepted interpretation of the (Special) Theory of Relativity assumes that (A) $t_1, t_1', t_2, t_2', \ldots$ as shown by clocks at points $P, P', Q, Q', \ldots$ in inertial systems S, S' at the instants (moments or simultaneously) when events $1, 2, \ldots$ happen at $P, P', Q, Q', \ldots$ satisfy the Lorentz transformations with the properties that (B) in general, if $t_1 = 0$, then $t_1' \neq 0$, if $t_2 = 0$, then $t_2' \neq 0$, if $t_2 = t_1$, then $t_2' \neq t_1'$, and vice versa, etc. The usual interpretation assumes further that (C) relative to S, S', t_1, t_1' are the times of the event 1, t_2, t_2' the times of the event 2, $t_2 - t_1$, $t_2' - t_1'$ the time intervals between the events 1 and 2, etc. We shall now discuss the statements (abbreviated hereafter as Sts.) (A)-(C). It is gratifying to know that some of the ideas expressed here are similar to those of Lorentz (1904), Poincaré (1904), Einstein (1905), Ives (1951), Fock (1964) and Janossy (1971), etc. A preliminary report of this paper is given in Nuthakki (1973). The usual interpretation with original references can be found, for example, in Møller (1952).

2. *Units and Synchronisation of Clocks*

The basis of the quantitative aspects of physical sciences is that physical quantities such as distances, time intervals, etc., are represented by numerical quantities times their respective units and compared with others of the same kind by their numerical quantities. Let the number of 'ticks' of a clock of an observer from one instant to another instant be t and let the duration between successive ticks be the unit second. Now, one can say that the time interval between the two instants is t times the second or simply t sec, the time interval between two other instants is t' sec$'$, etc., if the following condition is satisfied:

(1) The units of the observers are to be constant throughout the measurements.

Statements such as the former time interval is t/t' times the later time interval, etc., are correct if condition (1) and the following are satisfied:

(2) The units of the observers are to be the same throughout the measurements.

Denoting the time interval between the former two instants by T, the time interval between the later two instants by T', we have $T = t$ sec, $T' = t'$ sec$'$. It does not really matter whether one identifies the symbols in mathematical relations with T, T', etc., or t, t', etc., as long as the units sec, sec$'$, etc., are the same; otherwise one should make a distinction between T, T', and t, t', etc.

Whenever we refer to the measurements directly or indirectly, for example when we say that (1) and (2) are satisfied or not satisfied, we mean at least within the accuracy required in a given case. As usual, we assume that an event happens at an instant at a point. Thus, when we refer to an event, for example when we say at (or from) an event, we mean at (or from) the instant or point at which the event happens. However, when we refer to an instant, we have in mind an event or a set (or sets) of simultaneous events happening at the instant at a point or different points in the same or different systems. When we say the same event, we mean the same event happening at the same instant at the same point. Similarly, a rod is the same as regards its length if its length remains unaltered during the time interval under consideration.

Now, if all observers in all systems in a given case measure the time intervals of all events from one and the same instant, then, for the sake of brevity, one can simply call them times of events. Thus, when expressed in terms of numerical quantities times units, times of events mean the time intervals of events from one and the same instant. Hence, in order to measure, for example, times of events, it is necessary to satisfy one more requirement:

(3) The clocks of the observers are (understood) to be set to the same value zero effectively at the same instant (event or set of simultaneous events).

Let us note that (1-3) are not postulates, definitions, axioms, etc.; they are the most basic requirements (conditions) of physical sciences that have to be satisfied in representing physical quantities by numerical quantities times their respective units and comparing them with each other of the same kind by their numerical quantities. St. 3 is also the property (meaning) of synchronisation of clocks. Let the clocks at the origins O, O' in S, S' be set to zero at the instant when the event o happens at O, O'. Let the clocks at P, P', Q, Q' in S, S' be set to zero at the instant/instants when events p, p', q, q' happen at P, P', Q, Q' respectively. The events p, p', q, q' may be considered as happening at P, P', Q, Q' at the instant/instants of the arrival of the light signals at P, P', Q, Q' in the usual method of the settings of the clocks as we shall discuss later. Denoting the numerical values of the ticks and the constant units of the clocks at P, P' from the instant/instants when the events p, p' happen to the instant when the event 1 happens by t_{1p}, $t'_{1p'}$ and $\sec_{1p}$, $\sec'_{1p'}$, the time interval of the event 1 from the event p is given by t_{1p} $\sec_{1p}$, from the event p' by $t'_{1p'}$ $\sec'_{1p'}$. Similarly, the time interval of the event 2 from the event q is given by t_{2q} $\sec_{2q}$, from the event q' by $t'_{2q'}$ $\sec'_{2q'}$. [At the instant at which the event o happens, the clocks at P, P', Q, Q' must be showing, from the events p, p' q, q', some (known or unknown) values, say, t_{op}, $t'_{op'}$, t_{oq}, $t'_{oq'}$. Now, the time of the event 1 (from the event o) is given by t_{1p} $\sec_{1p}$ $-$ t_{op} $\sec_{op}$, $t'_{1p'}$ $\sec'_{1p'}$ $-$ $t'_{op'}$ $\sec'_{op'}$ where the two expressions are equal. The time of the event 2 is given by t_{2q} $\sec_{2q}$ $-$ t_{oq} $\sec_{oq}$, $t'_{2q'}$ $\sec'_{2q'}$ $-$ $t'_{oq'}$ $\sec'_{oq'}$ where the two expressions are equal.] If the events $o, p, p', \ldots$ happen at the same instant (St. 3), then the time of the event 1 is given by t_{1p} $\sec_{1p}$, $t'_{1p'}$ $\sec'_{1p'}$ where $t_{1p} \sec_{1p} = t'_{1p'} \sec'_{1p'}$, etc. If the units are the same, then $t_{1p} = t'_{1p'}$, etc. Now, for the sake of brevity, one may omit the units and the second subscripts and simply say that the time of the event 1 is $t_1 = t'_1$, the time of the event 2 is $t_2 = t'_2$, the time interval between the events 1 and 2 is $t_2 - t_1 = t'_2 - t'_1$, etc. Thus, unless (1)–(3) are satisfied, it is incorrect to say that relative to S, S', t_1, t'_1 are the times of the event 1, t_2, t'_2 the times of the event 2, $t_2 - t_1, t'_2 - t'_1$ the time intervals between the events 1 and 2, etc.; if (1)–(3) are satisfied, then t_1 must be equal to t'_1, t_2 to t'_2, $t_2 - t_1$ to $t'_2 - t'_1$, etc., anyway. (This is, of course, a way of knowing whether (1)–(3) are satisfied or not.) Thus, in any case, statements such as (C), concerning times of events, time intervals between events, simultaneous events, distances, velocities, etc., are incorrect. Hence, it is incorrect to use terms such as 'local time' (Lorentz, 1904; Poincaré, 1904), time values of events near A, time values of events near B, an 'A time', a 'B time' (especially when the clocks at the points A and B are not yet synchronised) (Einstein, 1905), clocked times (Ives, 1951), measures of the time of an event (Janossy, 1971), time in one frame not the same as time in another (Fock, 1964), proper time, relative time, curved time, space-time continuum etc.

The above considerations show the importance of (1)–(3). The fundamental question one should ask is whether (1)–(3) are satisfied or not. The usual interpretation assumes that the measuring instruments in S, S' are identical when they are placed side by side at rest relative to each other in an inertial system, say S. One should ask whether the rates, lengths, etc., of the measuring instruments are identical and unaltered when they are at rest in inertial systems at the same and different points and when they undergo accelerations in moving them to different positions in the same and different inertial systems. One should also ask whether the measuring instruments vary with measurements, observed processes, associated 'fields' of S, S', etc., whether the measurements change the very physical quantities of the processes under investigation, etc., and whether the observers in S, S' can measure the same process as required, etc. The usual interpretation gives no (satisfactory) answers for most of the above questions. We will come back to them later.

As regards the synchronisation of clocks, we have the following situation according to the usual interpretation: Let a light signal, departing from the origins O, O' at the same (instant) event o happening at O, O' when O, O' are in coincidence and the clocks at O, O' show $t = t' = 0$, arrive at the points R, R' in S, S' at the same (instant) event e happening at R, R' when R, R' are in coincidence. Because of the relative motion of S and S', the signal returns to O, O' at different (instants) events f, f' happening at O, O' when O, O' are separated and the clocks at O, O' show, say, $2t, 2t'$ respectively. Now, according to the usual interpretation, (i) the light signal travels in equal time interval t from O to R and R to O between the events o and e, and e and f, (ii) the light signal travels in equal time interval t' from O' to R' and R' to O' between the events o and e, and e and f', (iii) the time interval between the same events o and e is different relative to S, S', (iv) the same light signal travels with the same velocity between the same two (instants) events o and e relative to S, S', (v) the clocks at R, R' are said to be synchronised with the clocks at O, O' if they are set to the values t, t' respectively in effect at the (instant) event e, etc.

According to the usual interpretation, the clocks at the origins of the systems are set to zero at the same (instant) event o when the origins coincide (in the case of the special Lorentz transformations). The usual interpretation is silent on the fundamental question whether the clocks at the other points in S, S' are (understood to be) set to zero effectively at the same instant (event or set of simultaneous events) or not. If they are not, then the clocks are not synchronised. All methods and techniques of the synchronisation of clocks satisfy (3) if they are correct.

It is incorrect to say that by definition (Einstein, 1905), the 'time' taken by the light signal to travel from O to R is equal to the 'time' it requires to travel from R to O. This is not a matter to be decided by a definition; it is a matter to be decided by an experiment. If one cannot determine it experimentally, then one may state it as an assumption, but not as a definition. If the fact that the word time is placed in inverted commas implies any (systematic) errors in the time intervals between events, then one should specify their magnitudes and consequences. At any rate, there can only be one time interval between any two instants, such as the instants at which the events o and e happen. The time interval between the events e and f is different from the time interval between the events e and f' as the events f and f' happen at different instants. Moreover, it is just impossible for any ray of light or any body to move with the same velocity between the same two (instants) events relative to S, S'. After taking into account the above points, one may still assume that light requires equal time interval to travel from A to B and B to A by changing the fundamental assumptions of the above Sts. (i)–(v) as we shall see later.

If one assumes (A) and, in general, $t_1 \neq t'_1$ (t_1 sec $\neq t'_1$ sec$'$), etc. in accordance with the presently accepted interpretation, then one or more of (1)–(3) are not satisfied, at least to the extent of $t_1 \neq t'_1$, etc. As a result, (A) is incomplete and (C) is incorrect. The usual interpretation should be modified, at least to the following extent:

(A$'$) The readings $t_{1p}, t'_{1p'}, t_{2q}, t'_{2q'}$ of the clocks at the points P, P', Q, Q' in inertial systems S, S' at the events 1, 2 happening at P, P', Q, Q' from the events p, p', q, q' at which the clocks at P, P', Q, Q' are set to zero satisfy the Lorentz transformations with the properties that (B$'$) in general, if $t_{1p} = 0$, then $t'_{1p'} \neq 0$, if $t_{2q} = 0$, then $t'_{2q'} \neq 0$, if $t_{2q} = t_{1p}$, then $t'_{2q'} \neq t'_{1p'}$ and vice versa, etc. (C$'$) Instead of calling the times of the events 1, 2 relative to S, S', $t_{1p}, t'_{1p'}, t_{2q}, t'_{2q'}$ should be called by some other names such as the c-readings at the events 1, 2 from the events p, p', q, q'; instead of calling time intervals between the events 1 and 2 relative to S, S', $t_{2q} - t_{1p}$, $t'_{2q'} - t'_{1p'}$ may be called the c-differences between the events 1 and 2 from the events p, p', q, q'; instead of calling coordinates, distances, velocities, etc., one may use the terms r-readings, r-differences, 'rocities', etc. Thus, the usual condition for simultaneous events, viz., $t_{2q} = t_{1p}$ in S or $t'_{2q'} = t'_{1p'}$ in S' is incorrect as they are not the times of the events 1 and 2.

Even if one assumes that clocks are placed only at the origins instead of at the points where the events happen, we get similar conclusions. Now, the instants of the events 1, 2 as observed in S, S' are not known because of the inaccuracy involved in the determination of the velocities of the information signals of the events. In this case, $t_{ao}, t'_{a'o}, t_{bo}, t'_{b'o}$ satisfy the Lorentz transformations where the instants a, a' and b, b' are supposed to correspond to the two instants of the events 1 and 2, but they are not. As long as one assumes the assumptions such as those mentioned above, then the above conclusions, viz., times of events, time intervals between events, simultaneous events, distances, velocities, etc., are not measurable either in S or in S', at least to the extent of t_{1p} sec$_{1p} \neq t'_{1p'}$ sec$'_{1p'}$, etc., are inevitable. It is conceivable that some properties of nature are measurable and some are not, at least within the accuracy required in a given case. However, as an alternative we assume that time intervals, distances, etc., are measurable at least within the accuracy required in the case of the Lorentz transformations, but the usual interpretation is to be modified, as we shall see below. As a result, both (C) and (C$'$) are incorrect.

3. *Processes Moving Relative to a System*

So far we have concentrated on single events. A single event or isolated single events are not very useful in physics. We compare the development of physical processes. It is, of course, not necessary that every two events as observed in a system should belong to the same process. Two similar physical processes starting at different times and/or with different velocities from a point, say, the origin of a system can always be assumed to reach any two events as observed in the system. In order to do experiments in equal time intervals, it is also not necessary that the observers should do the experiments between the same two instants (events).

We make a fundamental distinction between the following two cases: (a) when physical processes are measured from one and the same system and/or the physical processes move relative to one system and are at rest relative to the other system, and (b) when the physical processes move relative to both systems. Let us note that the physical processes connected by certain relations that are obtainable from the Lorentz transformations are measured from the same system. Some (if not all) of these relations can also be deduced from other considerations. A surprisingly large number of relations come under the above class. Even in the case of such relations as $t = t_0\gamma$ where $\gamma = (1 - v^2/c^2)^{-1/2}$, one can say that in addition to v and the velocity of light c, t_0 sec and t sec are measured from the same system, say S for the time intervals of two similar processes, one at rest in S and one in motion with velocity v m/sec relative to S in reaching the same stage of development. One can interpret in a similar way the relation $L = L_0/\gamma$ connecting the lengths of two similar rods and the relation $m = m_0\gamma$ connecting the masses of two similar particles. These relations can also be interpreted in the following way:

Let an observer measure t_0 sec for the half-life of a radioactive substance using a clock when all of them are at rest in S. Let all of them be imparted a uniform velocity v m/sec relative to S along the positive x-axis of S. It is obvious that they undergo certain acceleration relative to S before attaining the uniform velocity v m/sec. Let the new system be called S'. Let the observers in S, S' measure t sec and t_0' sec$'$ for the half-life of the substance at rest in S' between the same two (instants) events. Now, according to the usual interpretation, (i) $t_o = t_o'$, (ii) $t = t_o'\gamma$, (iii) the half-life of the substance in S as measured in S is equal to the half-life of the substance in S' as measured in S', (iv) the half-life of the substance in S' between the same two (instants) events is different relative to S, S', (v) a clock in S' goes slow compared to the clocks in S by γ and (vi) vice versa, (vii) the clock at A' in S' lags behind the clock at A in S by γ between the same two (instants) events of its departure from and arrival at the point A in S where the time periods of its acceleration and deceleration are negligible compared to the time intervals of its uniform motion. Statements such as the above give rise to the famous clock-paradox (see, for example, Møller, 1952).

As mentioned before, statements such as (iv) are, of course, incorrect. If the effects referred to in (v) and (vi) are only apparent, i.e., if measurements, accelerations, etc., have no effect on the rates of the clocks, then (vii) is incorrect. If the rates of the processes in S' are unaffected when they are accelerated from S in attaining the uniform velocity v m/sec, then (iii) is correct. If the measurements (and not accelerations, etc.) of the processes in S' from S change the rates of the processes in S' (or S) by γ and vice versa, then (i–iii, v–vii) are correct. In this case, when measured sec = sec$'/\gamma$ and when not measured sec = sec$'$. The St. (iv) should be changed accordingly. However, we explain the above cases in the following way:

We make a distinction between (macroscopic) systems from which one can make measurements and (microscopic) systems from which one cannot for the simple reason that the measuring instruments cannot be placed on the latter. We denote S' as considered here by s to signify its acceleration from or association with S. We assume that a large body like earth possesses its associated aether, and the rates of the processes and the lengths of the rods in s change by γ and $1/\gamma$ respectively when they are accelerated from S to attain the uniform velocity v m/sec. The rates of the clocks and the radioactive substance in s change by the same factor γ and hence (i), (ii), (v) and (vii) are correct. As the units in S and s are different, (iii) and (iv) are incorrect. Instead of (vi), we say that a clock when accelerated from S' goes slow compared to the clocks in S'. Unless one explains in a manner similar to the above, the clock-paradox cannot be resolved.

So far, we have considered the case (a). Let us remember that almost all the measurements supporting the consequences of the Lorentz transformations came in each case from two or more physical processes as measured from the same system. No two relatively moving observers in two inertial systems have yet measured independently under similar circumstances the quantities $X = (x_1, x_2; t_1, t_2; u_1, u_2; \ldots)$ and $X' = (x'_1, x'_2; t'_1, t'_2; u'_1, u'_2; \ldots)$ for distances, time intervals, velocities, etc., of the same process that moves relative to both the observers at the same two (instants) events 1 and 2 of the process and verified the properties of the Lorentz transformations. Hence, the claim that the usual interpretation is verified is incorrect. It may be that the consequences of the Lorentz transformations are, in general, valid for physical processes as observed in each case from the same system; to that extent only the Lorentz transformations are so far verified. We may say that the Lorentz transformations are just mathematical devices that enable one to obtain the laws of physical processes as observed in any one inertial system. It means that the case (b) is excluded. However, the usual interpretation assumes that the Lorentz transformations are also valid for the general case (b). Assuming the same, let us see what happens.

4. *Processes Moving Relative to Two Systems*

We now assume that the usual assumption (A), in general, is altogether incorrect. As a result, the usual and modified interpretations (A, C and A$'$, C$'$) are incorrect. The usual St. (C) is incorrect in any case. We assume the following:

In each case, at least two similar physical processes are generally involved with the Lorentz transformations, taking in general different time intervals with different events at different instants in reaching the same stage of development.

The principle of relativity is quite flexible and can absorb the above assumption. In stating the principle of relativity, Fock (1964) supposes corresponding physical processes, but the fundamental assumption that the physical processes generally take different time intervals with different events at different instants in reaching the same stage of development is not introduced. Like almost all scientists, he believes in the presently accepted concepts, such as the same events that are simultaneous relative to a system are not so relative to other systems, etc. If one believes in such incorrect concepts, then the assumption of corresponding processes is quite different and plays quite a different role to the one we are concerned with.

In case (a) or (b), the observers in an inertial system can determine whether events are simultaneous or not, but the measurements of the same events by observers in other systems are, in general, ruled out, at least from the considerations of the Lorentz transformations. If the observers in S measure a set of simultaneous events happening at an instant and the observers in S' measure another set of simultaneous events happening at the same instant, then both sets of simultaneous events are simultaneous.

One of the fundamental properties of the Lorentz transformation is that (D) $\Delta t^2 \gtreqless \Delta t'^2$ for $\Delta x^2 + \Delta y^2 + \Delta z^2 \gtreqless \Delta x'^2 + \Delta y'^2 + \Delta z'^2$ where $\Delta t = (t_2 - t_1)$, etc. In order to explain on the basis of the usual interpretation the properties (B), (D), etc., and the Sts. (i)–(vii), etc., one may say that the clocks are set effectively to zero once only for all the experiments, but the rates, lengths, etc., of the measuring instruments in S, S' are fixed and kept constant throughout all the experiments or they vary from one experiment to another. In the former case, the effects of the Lorentz transformations are only apparent. Such an assumption cannot explain the observed data. In order to consider the latter case, let us take the general case when a physical process moves with velocities u, u' relative to S, S'. One may say that the measurements of the process from S, S' change the rates, lengths, etc., of the measuring instruments in S, S' respectively. Such an assumption is most unlikely to be correct and at any rate cannot satisfactorily explain the properties of the Lorentz transformations and the observed data. The usual interpretation insists that the observers in S, S' measure the physical quantities of the same process between the same two (instants) events of the process. Thus, an implicit assumption of the usual and modified interpretations is that the observers in S, S' can measure the physical quantities of the same process between the same two (instants) events of the process independently, and under similar circumstances, without changing the very physical quantities of the process under investigation and without interfering with the measurements of each other. This may not be possible especially in the case of the microscopic systems.

If the measurements of the process from S, S' change the rate of the process (by γ_1, γ_2 where $\gamma_1 = (1 - u^2/c^2)^{-1/2}$, $\gamma_2 = (1 - u'^2/c^2)^{-1/2}$), then the observers in S, S' should make measurements in the absence of the measurements of each other, i.e., at different instants. In this case, the process, when measured from S, S', takes different time intervals with different events at different instants in reaching the same stage of development. Let us remember that each experiment consists of at least two measurements. One may say that the first measurement from, say, S makes the process align relative to S and in so doing the rate of the process changes by γ_1. The second measurement completes the experiment. One may say that the aether of S and not the measurements from S changes the rate of the process. This may or may not be connected with the acceleration of the process from S. As mentioned before, we assume that the acceleration of the process from S changes its rate by γ_1. A process accelerated to a higher velocity takes more time and moves a greater distance than a process accelerated to a lesser velocity in reaching the same stage of development.

Let there be two radioactive substances s, s' at rest in S, S' respectively with $2n$ particles in each. Let the substances be accelerated from S, S' to attain certain uniform velocities relative to S, S'. Let the observers in S measure the velocity u m/sec and the half-life t sec for the substance s when n particles decay in a distance x m in S. Similarly, let the observers in S' measure the corresponding quantities u', t', x' for the substance s' when n particles decay. Let the clocks accelerated from S, S' to follow the substances s, s' show τ $\sec_o$, τ $\sec_o'$ for the half-lives respectively. It is, of course, not necessary to have clocks to follow the substances. The radioactive processes may themselves be considered as clocks. Let us remember that the numerical quantity τ corresponds to the same stage of development, viz., the decay of n particles in both substances. Now, we have $t = \gamma_1 \tau$, $t' = \gamma_2 \tau$, $x = ut$, $x' = u't'$, from which one can obtain the Lorentz transformations. The units $\sec_o \neq \sec_o'$ and t and t' correspond to different time intervals with different events at different instants. In the general case, when the processes move relative to both S, S' ($u \neq 0$, $u' \neq 0$), i.e., when the relationship of s to S is similar to the relationship of s' to S' in motion (as regards the aether of S, S', etc.), then t, t' (and x, x') may be considered as expressed in the same units. One can easily understand the case $u' = 0$ or $u = 0$ in more than one way as discussed before.

Acknowledgments

The author is thankful to Professor G. Costa and Dr. Nuthakki, Pratibha, for reading an earlier version of the paper and making useful comments. Most of the present work was done at Boston College, Chestnut Hill, U.S.A.

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